OPTIMIZATION OF TRANSMISSION SCHEDULES IN CAPTURE-BASED WIRELESS NETWORKS

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ABSTRACT

In this paper, we study transmission strategies in multiple-source, multiple-destination wireless networks. Each source is within communication range of its intended destination. However, packets can cause interference at other destinations. The source nodes are first divided into groups, based on the intended destination of their packets. We initially assume that each group operates according to its own local TDMA schedule, independently of the other groups. Our primary performance measure is throughput, which we define to be the average number of packets that are successfully received per intended destination per time slot. We then develop mathematical methods for evaluating the performance of the network for a given arbitrary schedule. Our results show the impact of schedule, channel fading, receiver noise, and interference on network performance. Next, we exploit network and channel state information such as topology and channel conditions to optimize network performance. In particular, for given channel statistics and topology configurations, we determine a schedule that maximizes the throughput. We show that the network performance can be significantly improved when there is coordination among the groups in the network.

1. INTRODUCTION

We study a multiple-source, multiple-destination network, in which each of the K source nodes transmits to a designated one of the N destinations. The network operates in the presence of detrimental effects such as channel fading, receiver noise, and other-user interference (i.e., a node's transmission may cause interference at non-intended destinations). An example is a wireless sensor network, which consists of K sensor nodes transmitting data to N collecting centers. Fig. 1 shows such a network in which $K = 15$ sources transmit to $N = 3$ destinations.

In this paper, we study a simple method for accomplishing the transmissions between the source nodes and their destinations. First, we organize the network nodes into some logical groups, which are defined based on the designated destination. Each group then operates according to a local TDMA protocol, concurrently and independently of the other groups. Our method can be extended to include protocols other than TDMA. A schedule is defined as a rule that governs how the network nodes operate. We then develop methods for evaluating the throughput performance of this "parallel" method for an arbitrary schedule. Our results shows the impact of channel model, receiver noise, and interference on network performance.

Next, we aim to improve the network performance by exploiting network information such as scheduling, topology, and channel conditions. We show that, in many cases, the network performance is improved significantly if the nodes operate according to an optimized schedule. Our model is related to the TDMA approach in [8], which addresses relay strategies in a single-destination network that operates in an environment that is free of other-user interference. In contrast, our model deals with a network that has multiple destinations and operates in an environment that includes other-user interference. We address the issue of throughput performance and optimization of transmission schedules under a heavy-traffic model, in which each source node always has traffic to transmit. Thus, it is not meaningful to compare our model to other protocols such as CSMA [2, 9], CDMA [7, 10], or 802.11 that make different assumptions about available resources and capabilities.
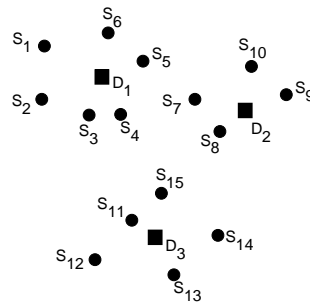

Fig. 1 A wireless network with 15 sources
and 3 destinations

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2. NETWORK MODEL AND ASSUMPTIONS

We consider a stationary wireless network that has K source nodes, denoted by $S_1, S_2, \cdots, S_K$, that transmit their traffic to N destinations, denoted by $D_1, D_2, \cdots, D_N$, $N \leq K$. Each source node transmits to only one particular intended destination (but will cause interference at the other destinations). Logically, we can partition the source nodes into N groups $G_1, G_2, \cdots, G_N$, where G_i is the set of source nodes that transmit to destination D_i. For example, let us revisit the 15 source nodes in Fig. 1, and further assume that (i) $S_1, S_2, \cdots, S_6$ are intended for D_1, (ii) S_7, S_8, S_9, S_{10} are intended for D_2, and (iii) $S_{11}, S_{12}, \cdots, S_{15}$ are intended for D_3. The sources can then be partitioned into the 3 groups as shown in Fig. 2.
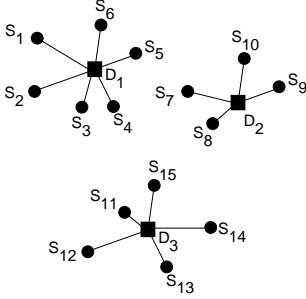

Fig. 2 The 15 sources are partitioned into 3 groups: $G_1 = \{S_1, \cdots, S_6\}$, $G_2 = \{S_7, \cdots, S_{10}\}$, and $G_3 = \{S_{11}, \cdots, S_{15}\}$

We assume the following:

- The nodes, whose locations are known and fixed, are equipped with omnidirectional antennas.
- Each destination can receive at most one successful transmission at a time. However, it is possible to extend this analysis to nodes with multiple reception capability.
- Each source node can communicate directly with its destination, i.e., it does not rely on other nodes to relay its traffic. However, our model can be extended to include multi-hop communication by letting some nodes be both sources and destinations, i.e., such nodes act as relay nodes.
- Each source node always has traffic to transmit, i.e., its transmission queue is never empty.
- Time is divided into slots. The traffic is expressed in terms of fixed-size packets such that it takes one time slot to transmit one packet. A frame consists of M_{frame} consecutive time slots.
- Our primary performance measure is throughput, which is the average number of packets that are successfully received by each intended destination in a time slot. We do not address issues such as time delays and stability analysis in this paper.
- The propagation delay among the nodes in the network is negligible.
- Nodes transmit according to a schedule, i.e., a node can transmit only in an assigned time slot. We require that each source node transmits at least once in each frame, and that the schedule repeats from frame to

frame. Thus, it is sufficient to study the performance in any one frame.

Definition 1. A *schedule* is a tuple

$$(H_1, H_2, \cdots, H_{M_{\text{frame}}})$$

where H_k is the set of source nodes that simultaneously transmit in time slot k.

Thus, a schedule is completely determined when the frame length M_{frame} and the sets H_k are determined, $k = 1, 2, \cdots, M_{\text{frame}}$. Clearly, the number of all possible schedules is very large for a general network. Later in the paper, we will impose some structure on the schedules to make the problem more tractable.

The network is operated based on the principle of power capture, i.e., a packet is successfully received, even in the presence of interference and noise, as long as its signal-to-interference-plus-noise ratio (SINR) exceeds a given threshold [3, 4, 12]. More precisely, suppose that we are given a set H of source nodes that transmit in the same time slot. Let $P_{\text{rx}}(S, D)$ be the signal power received from node S at node D (when node S transmits), and let $\text{SINR}(S, D)$ be the SINR determined by node D due to the transmission from node S, i.e.,

$$\text{SINR}(S, D) = \frac{P_{\text{rx}}(S, D)}{P_{\text{noise}}(D) + \sum_{U \in H \setminus \{S\}} P_{\text{rx}}(U, D)}$$

where $P_{\text{noise}}(D)$ denotes the receiver noise power at node D. We assume that a packet transmitted by S is successfully received by D if and only if it is intended for D and

$$\text{SINR}(S, D) > \beta_D \tag{1}$$

where $\beta_D \geq 0$ is a threshold at node D, which is determined by application requirements and the properties of the network [11]. When $\beta_D < 1$ (e.g., in spread-spectrum networks), it is possible for 2 or more transmissions to satisfy (1) simultaneously [5].

The wireless channel is subject to fading, as described below. Let $P_{\text{tx}}(i)$ be the transmit power at node i, and $r(i, j)$ be the distance between nodes i and j. When node i transmits, the power received by node j is modeled by

$$P_{\text{rx}}(i, j) = A(i, j)g(i, j)$$

where $A(i, j)$ is a random variable that incorporates the channel fading. We refer to $g(i, j)$ as the "received power factor," which depends on $r(i, j)$ and $P_{\text{tx}}(i)$. For far-field communication (i.e., when $r(i, j) \gg 1$), we have

$$g(i, j) = P_{\text{tx}}(i)r(i, j)^{-a} \tag{2}$$

where a is the path-loss exponent whose typical values are between 2 and 4. A simple approximate model for both near-field (i.e., when $r(i, j) < 1$) and far-field communication is

$$g(i, j) = P_{\text{tx}}(i)[r(i, j) + 1]^{-a} \tag{3}$$

where the term $r(i, j) + 1$ is used to ensure that $g(i, j) \leq P_{\text{tx}}(i)$. Under Rayleigh fading, it is well known that $A(i, j)$ is exponentially distributed [6].

Our goal is to study methods for accomplishing the communication between the sources and destinations, and to analytically evaluate the resulting performance. Under the well-known (global) TDMA method, each source node is given a turn to transmit, i.e., there is exactly one transmission in each time slot. Thus, no other-user interference is present. In this paper we propose a "parallel" approach, as described in the following, under which the (local) TDMA groups simply operate in parallel.

Recall that the source nodes are partitioned into N groups $G_1, G_2, \cdots, G_N$, where G_i is the set of source nodes that are intended for destination D_i. We assume that the nodes in each group operate according to a "local" TDMA discipline that involves only members of that group. Thus, there are N transmissions in each slot, each of which is intended for a different destination. Note that the groups can operate according to a randomly chosen schedule. There is then no coordination among the different groups, i.e., they operate simultaneously and independently of each other. However, as seen later, when these groups coordinate among themselves to obey an optimal schedule, the throughput performance can be significantly improved.

Let us revisit Fig. 2, which shows the network with $N = 3$ groups. According to our rule, there are 3 transmissions in each time slot. An example is shown in Fig. 3, which shows 3 transmissions ($S_1 \rightarrow D_1$, $S_{10} \rightarrow D_2$, and $S_{15} \rightarrow D_3$) in some time slot. Note that each transmission will cause interference at all of the unintended destinations.
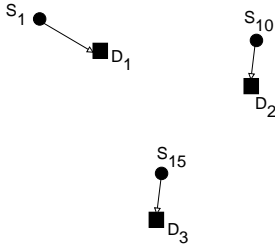

Fig. 3 Under the parallel method, sources S_1, S_{10} and S_{15} transmit in the same time slot

3. THROUGHPUT EVALUATION

For a given schedule, our performance measure is the average number of packets that are successfully received in a time slot. For a given time slot k, define $C_H^k(S, D)$ to be the probability that a packet from source node S is successfully received by its destination node D, given that all the nodes in H simultaneously transmit in this time slot. Let $C_{\text{success}}(k)$ be the average total number of successful transmissions in time slot k.

Recall that H_k denotes the set of source nodes that transmit in time slot k (see Definition 1). Under the parallel method, there are N transmissions in each time slot, i.e., $|H_k| = N$. We then have

$$C_{\text{success}}(k) = \sum_{S \in H_k} C_{H_k}^k(S, D^S) \qquad (4)$$

where D^S denotes the destination of S.

We now define throughput T to be the average number of packets that are successfully received per intended destination per time slot. Because each destination can receive at most one packet in a time slot, T is also the probability that a packet is successfully received by its intended destination in a time slot. Recall that there are M_{frame} time slots in a frame, and there are N transmissions in each time slot. Thus, the total number of transmissions in each frame is NM_{frame}. The throughput is then

$$T = \frac{1}{NM_{\text{frame}}} \sum_{k=1}^{M_{\text{frame}}} C_{\text{success}}(k)$$

which, from (4), becomes

$$T = \frac{1}{NM_{\text{frame}}} \sum_{k=1}^{M_{\text{frame}}} \sum_{S \in H_k} C_{H_k}^k(S, D^S) \qquad (5)$$

The following result, which is equivalent to the result in [1], gives the exact formula for $C_{H_k}^k(S, D^S)$ for the case of Rayleigh fading, which depends on the receiver noise, channel fading, receiver threshold, and other-user interference.

Theorem 1. Let t be a time slot and let H be the set of nodes that simultaneously transmit in t. Let k be a receiving node (e.g., a destination). Suppose that the fading between a transmitting node i and the receiving node k is modeled as a Rayleigh random variable Y_i with parameter $v(i, k)$. For $i \neq j$, assume that Y_i and Y_j are independent. Let $g(i, k)$ denote the received power factor, which depends on the distance and the transmit power, e.g., $g(i, k) = P_{\text{tx}}(i)r(i, k)^{-a}$ or $g(i, k) = P_{\text{tx}}(i)\left[r(i, k) + 1\right]^{-a}$. Given that all the nodes in H simultaneously transmit in time slot t, the probability that a packet from node i is successfully received by k is

$$C_H^t(i, k) = \exp\left(-\frac{\beta_k P_{\text{noise}}(k)}{v(i, k)g(i, k)}\right)$$
$$\times \prod_{j \in H \setminus \{i\}} \left[1 + \beta_k \frac{v(j, k)g(j, k)}{v(i, k)g(i, k)}\right]^{-1}$$

where β_k and $P_{\text{noise}}(k)$ are the required SINR threshold and the receiver noise power at node k, respectively.

In particular, we have

$$C_{\{i,j\}}^t(i, k) = \exp\left(-\frac{\beta_k P_{\text{noise}}(k)}{v(i, k)g(i, k)}\right)$$
$$\times \left[1 + \beta_k \frac{v(j, k)g(j, k)}{v(i, k)g(i, k)}\right]^{-1}$$

and

$$C_{\{i\}}^t(i, k) = \exp\left(-\frac{\beta_k P_{\text{noise}}(k)}{v(i, k)g(i, k)}\right)$$

which is the the probability that a packet from node i is successfully received by k, when there is no interference from another node.

Proof. Let F_X and f_X denote the cdf and pdf of random variable X. To simplify the notation, we define $P_i = P_{rx}(i,k)$, $P_{noise} = P_{noise}(k)$, $v_i = v(i,k)$, $g_i = g(i,k)$, and $\beta = \beta(k)$. Because $\text{SINR}(i,k) = \frac{P_i}{\sum_{j \in H \setminus \{i\}} P_j + P_{noise}}$, we have

$$\Pr\{\text{SINR}(i,k) \le \beta\}$$
$$= \Pr\left\{ P_i \le \beta\left[\sum_{j \in H \setminus \{i\}} P_j + P_{noise}\right]\right\}$$
$$= \int_0^\infty \cdots \int_0^\infty \Pr\left\{ P_i \le \beta\left[\sum_{j \in H \setminus \{i\}} x_j + P_{noise}\right]\right\}$$
$$\times \prod_{j \in H \setminus \{i\}} f_{P_j}(x_j) dx_j$$
$$= \int_0^\infty \cdots \int_0^\infty F_{P_i}\left(\beta\left[\sum_{j \in H \setminus \{i\}} x_j + P_{noise}\right]\right)$$
$$\times \prod_{j \in H \setminus \{i\}} f_{P_j}(x_j) dx_j \tag{6}$$

Because of fading, we have

$$P_i = A_i g_i \tag{7}$$

where A_i is a random variable. From (7), it can be shown that

$$F_{P_i}(x) = F_{A_i}(x/g_i) \tag{8}$$

The pdf of P_i is the derivative (8), i.e.,

$$f_{P_i}(x) = f_{A_i}(x/g_i)/g_i \tag{9}$$

Let Y_i be the Rayleigh random variable, i.e., $f_{Y_i}(x) = \frac{x}{v_i}\exp(-x^2/(2v_i))$, $x \ge 0$, where $v_i = E(Y_i^2)/2$. Under the Rayleigh fading, we have $A_i = Y_i^2$. It then can be shown that A_i is exponentially distributed with mean v_i, i.e.,

$$f_{A_i}(x) = \exp(-x/v_i)/v_i, \quad x \ge 0 \tag{10}$$

and

$$F_{A_i}(x) = 1 - \exp(-x/v_i), \quad x \ge 0 \tag{11}$$

Substituting (10) into (9), and (11) into (8), we have

$$F_{P_i}(x) = 1 - \exp\left(-x/(v_i g_i)\right) \tag{12}$$

and

$$f_{P_i}(x) = \exp\left(-x/(v_i g_i)\right)/(v_i g_i) \tag{13}$$

Substituting (12) and (13) into (6), we have

$$\Pr\{\text{SINR}(i,k) \le \beta\}$$
$$= \int_0^\infty \cdots \int_0^\infty \left[1 - \exp\left(-\frac{\beta\left[\sum_{j \in H \setminus \{i\}} x_j + P_{noise}\right]}{v_i g_i}\right)\right] \times$$
$$\prod_{j \in H \setminus \{i\}} \frac{1}{v_j g_j}\exp\left(-\frac{x_j}{v_j g_j}\right) dx_j$$
$$= 1 - \int_0^\infty \cdots \int_0^\infty \exp\left(-\frac{\beta\left[\sum_{j \in H \setminus \{i\}} x_j + P_{noise}\right]}{v_i g_i}\right) \times$$
$$\prod_{j \in H \setminus \{i\}} \frac{1}{v_j g_j}\exp\left(-\frac{x_j}{v_j g_j}\right) dx_j$$

Thus, $C_H^t(i,k)$
$$= \Pr\{\text{SINR}(i,k) > \beta\}$$
$$= \int_0^\infty \cdots \int_0^\infty \exp\left(-\frac{\beta\left[\sum_{j \in H \setminus \{i\}} x_j + P_{noise}\right]}{v_i g_i}\right) \times$$
$$\prod_{j \in H \setminus \{i\}} \frac{1}{v_j g_j}\exp\left(-\frac{x_j}{v_j g_j}\right) dx_j$$
$$= \exp\left(-\frac{\beta P_{noise}}{v_i g_i}\right)\int_0^\infty \cdots \int_0^\infty \prod_{j \in H \setminus \{i\}}\exp\left(-\frac{\beta x_j}{v_i g_i}\right) \times$$
$$\prod_{j \in H \setminus \{i\}} \frac{1}{v_j g_j}\exp\left(-\frac{x_j}{v_j g_j}\right) dx_j$$
$$= \exp\left(-\frac{\beta P_{noise}}{v_i g_i}\right)\int_0^\infty \cdots \int_0^\infty \prod_{j \in H \setminus \{i\}} \frac{1}{v_j g_j} \times$$
$$\exp\left(-x_j\left[\frac{\beta}{v_i g_i} + \frac{1}{v_j g_j}\right]\right) dx_j$$
$$= \exp\left(-\frac{\beta P_{noise}}{v_i g_i}\right) \times$$
$$\prod_{j \in H \setminus \{i\}}\int_0^\infty \frac{1}{v_j g_j}\exp\left(-x_j\left[\frac{\beta}{v_i g_i} + \frac{1}{v_j g_j}\right]\right) dx_j$$
$$= \exp\left(-\frac{\beta P_{noise}}{v_i g_i}\right)\prod_{j \in H \setminus \{i\}} \frac{1}{v_j g_j}\left(\frac{\beta}{v_i g_i} + \frac{1}{v_j g_j}\right)^{-1}$$
$$= \exp\left(-\frac{\beta P_{noise}}{v_i g_i}\right)\prod_{j \in H \setminus \{i\}}\left(1 + \beta\frac{v_j g_j}{v_i g_i}\right)^{-1} \quad \square$$

Remark 1. The above throughput T has the form of summation. It may also be of interest to consider a product-form measure of throughput, which is defined by

$$T_{prod} = \left(\prod_{k=1}^{M_{frame}} C_{success}(k)\right)^{\frac{1}{NM_{frame}}}$$

where $C_{success}(k) = \prod_{S \in H_k} C_{H_k}^k(S, D^S)$. The product-form throughput, which is appropriate for a model that encourages fairness among the nodes in the network, is not addressed in this paper.

Remark 2. For a given schedule, we can analytically compute the throughput T in (5). The computation of T requires a double sum that adds the $M_{frame}N$ terms of the form $C_{H_k}^k(S, D^S)$, where M_{frame} is the frame length and N is the number of destinations. The computation of $C_{H_k}^k(S, D^S)$ in turn requires a product of N terms (by Theorem 1). The overall computational complexity for computing T is then $O(M_{frame}N^2)$, which, for a given value of N, is minimized when M_{frame} is minimized. Thus, it is desirable to minimize M_{frame}.

Remark 3. Recall that, under the TDMA method, there is exactly one transmission in each time slot. Thus, there is no other-user interference. The throughput T for the parallel method is given in (5). Similarly, it can be shown that the throughput for the TDMA method is

$$T_{TDMA} = \frac{1}{KN}\sum_{i=1}^K C_{\{S_i\}}^i(S_i, D^{S_i})$$

where D^{S_i} denotes the destination of source S_i. We must have $T_{TDMA} \le 1/N$. This upper bound is achieved under the ideal condition $P_{noise}(D^{S_i}) = 0$, for all i.

4. SCHEDULE SPECIFICATION AND OPTIMIZATION FOR THE PARALLEL METHOD

Recall that we define a schedule in terms of a frame (Definition 1). Each frame has M_{frame} time slots. Also, in each time slot, there are N transmissions, each of which is intended for a different destination.

To ensure fairness among the source nodes that belong to the same group, we require that they have the same number of transmissions in each frame. For group G_i, this number is denoted by h_i. However, different groups may have different number of transmissions, i.e., we may have $h_i \neq h_j$ for some $i \neq j$. Let m_i be the number of source nodes in group i, i.e., $m_i = |G_i|$. We must have

$$h_i = \frac{M_{\text{frame}}}{m_i} \qquad (14)$$

From Remark 2, to simplify the computation of the throughput T in (5), we now require that the frame length M_{frame} be minimum. From (14), M_{frame} must be a common multiple of $m_1, m_2, \cdots, m_N$. Thus, M_{frame} is minimized only if it is the *least* common multiple (LCM) of $m_1, m_2, \cdots, m_N$, i.e.,

$$M_{\text{frame}} = \text{LCM}(m_1, m_2, \cdots, m_N) \qquad (15)$$

Let e denote the number of possible schedules. Using a combinatorial argument, it can be shown that

$$e = \prod_{i=2}^{N} \frac{M_{\text{frame}}!}{(h_i!)^{m_i}} \qquad (16)$$

To summarize, we can compute the throughput T in (5) for each of the e schedules. Thus, our model and formulation naturally lead to the following schedule optimization problem: Find an optimal schedule that maximizes the throughput T.

Example 1. Let us consider a wireless network with $K = 7$ sources and $N = 3$ destinations. Assume that (i) S_1 and S_2 are intended for D_1, (ii) S_3, S_4, and S_5 are intended for D_2, and (iii) S_6 and S_7 are intended for D_3. Thus, the sources are partitioned into $G_1 = \{S_1, S_2\}$, $G_2 = \{S_3, S_4, G_5\}$, and $G_3 = \{S_6, S_7\}$. The cardinalities of the groups are $m_1 = 2$, $m_2 = 3$, $m_3 = 2$, and hence $M_{\text{frame}} = \text{LCM}(2, 3, 2) = 6$. Two examples of schedules are shown below.

Time slot	1	2	3	4	5	6
Schedule 1	S_1	S_2	S_1	S_2	S_1	S_2
	S_3	S_4	S_5	S_3	S_4	S_5
	S_6	S_7	S_6	S_7	S_6	S_7
Schedule 2	S_1	S_1	S_1	S_2	S_2	S_2
	S_3	S_3	S_4	S_4	S_5	S_5
	S_6	S_6	S_7	S_7	S_6	S_7

From Definition 1, each schedule is specified by the tuple $(H_1, H_2, \cdots, H_6)$. For Schedule 1, we have $H_1 = \{S_1, S_3, S_6\}, H_2 = \{S_2, S_4, S_7\}, \cdots, H_6 = \{S_2, S_5, S_7\}$. For Schedule 2, we have $H_1 = \{S_1, S_3, S_6\}, H_2 = \{S_1, S_3, S_6\}, \cdots, H_6 = \{S_2, S_5, S_7\}$. In each frame of 6 slots, each member of G_1 transmits 3 times, each member of G_2 transmits 2 times, and each member of G_3 transmits 3 times, i.e., $h_1 = 3$, $h_2 = 2$, and $h_3 = 3$. Using (16), the number of schedules is

$$e = \frac{6! \; 6!}{(2!)^3 \; (3!)^2} = 600$$

Remark 4. Consider the special case where all the groups have the same size. We then have $m_i = |G_i| = K/N$, $M_{\text{frame}} = K/N$, and $h_i = 1$. Substituting these into (16), the number of schedules for this special case is

$$e = \left(\frac{K}{N}!\right)^{N-1} \qquad (17)$$

5. PERFORMANCE EVALUATION

In this section, we compare the throughput performance, by numerical examples, for both the TDMA method (see Remark 3) and our parallel method. However, our focus is on the parallel method, for which we show that further improvement in performance is possible with schedule optimization. We also show the impact of channel conditions, receiver noise level, other-user interference, network topology, and schedules on performance. We assume the following:

- The path-loss exponent is $a = 3$.
- The receiver threshold and the receiver noise power are the same at each destination D, i.e., we can write $\beta_D = \beta$ and $P_{\text{noise}}(D) = P_{\text{noise}}$.
- The wireless channel is subject to Rayleigh fading with Rayleigh parameter $v(i, j) = 1$.
- The received power factor is given by (3), i.e., $g(i, j) = P_{\text{tx}}(i)[r(i, j) + 1]^{-a}$.
- The transmit power is $P_{\text{tx}}(S) = 1$ for all sources S.

We now study a stationary wireless network as shown in Fig. 4, which has $N = 4$ destinations and $K = 12$ sources. The 4 destinations are located at the vertices of a 10×10 square. We assume that sources S_{3i-2}, S_{3i-1}, and S_{3i} are intended for destination D_i, and they are located randomly in the circle centered at D_i and of radius R, $1 \leq i \leq 4$. In particular, S_1, S_2, and S_3 are intended for D_1, and they are located randomly in the circle centered at D_1 and of radius R.

Note that R determines the distribution of the nodes that are associated with each particular destination. However, a node's transmission will cause interference outside these circles. As R increases, the degree of other-user interference increases.
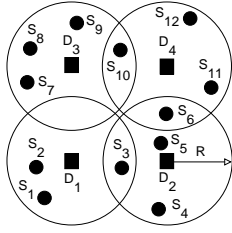
Fig. 4 A wireless network with 12 sources
and 4 destinations

The throughput for the TDMA method, T_{TDMA}, is given in Remark 3. The throughput T for the parallel method is given in (5). To specify the parallel method, we need to determine the groups and the schedules. Our assumptions imply that $G_1 = \{S_1, S_2, S_3\}$, $G_2 = \{S_4, S_5, S_6\}$, $G_3 = \{S_7, S_8, S_9\}$, and $G_4 = \{S_{10}, S_{11}, S_{12}\}$. In this example, all 4 groups have the same size of 3, i.e., $m_1 = m_2 = m_3 = m_4 = 3$.

From Remark 4, the frame lengh is $M_{\text{frame}} = 3$, and the number of all possible schedules is $e = (3!)^3 = 216$, which is small enough for us to use exhaustive search to find an optimal schedule. We are interested in the following 3 forms of the throughput T given in (5) for the parallel method:

- The maximum throughput, $T_{\max}$, which is produced by the best schedules.
- The minimum throughput, $T_{\min}$, which is produced by the worst schedules.
- The average throughput, T_{ave}, which is obtained by averaging the throughput values produced by the e schedules. We must have $T_{\min} \leq T_{\text{ave}} \leq T_{\max}$.

Under the parallel method, in order for the nodes in the network to obey a particular schedule, they must coordinate among themselves to meet the specifications of the schedule. Clearly, it is desirable for the network to operate according to one of the best schedules (that yield the maximum throughput $T_{\max}$) and to avoid poorly performing schedules. Note that the simplest form of distributed implementation would be to use randomly chosen schedules, in which there is no coordination among the groups. The long-term throughput performance under such random schedules is then estimated by T_{ave}. As seen in the following, even these random schedules can significantly outperform the TDMA method in most cases.

Recall that the source nodes that are intended for the same destination are located in the circle of radius R. The results are shown in Figs. 5 (for $R = 5$), 6 (for $R = 10$), and 7 (for $R = 20$). For each case, we show the performance for $P_{\text{noise}} = 0$ and $P_{\text{noise}} = 0.001$. Our results are evaluated for different values of the threshold β. Note that smaller β results in higher packet throughput but lower bit rate per transmission. The issue of translating from packet throughput into bit rate is addressed in [11].

From Figs. 5 through 7, for most cases, the throughput performance for the TDMA method (indicated by T_{TDMA}) is much lower than that of our parallel method (indicated by $T_{\max}$, T_{ave}, and $T_{\min}$). In fact, our results show that the TDMA method works best only if

$P_{\text{noise}} = 0$ and β is sufficiently large (i.e., $\beta \geq 5$ for $R = 10$ and $\beta \geq 2$ for $R = 20$). Thus, in the following we focus our discussion on the parallel method.

First, consider Fig. 5 (for $R = 5$). Under the parallel method, as expected, throughput decreases when the threshold β increases. Also, throughput decreases when the receiver noise level P_{noise} increases. We observe that the variation in throughput (indicated by $T_{\max}/T_{\min}$) for different schedules is negligible for $\beta \leq 1$. The variation is only slightly larger for $\beta \geq 2$. Thus, schedule optimization yields only modest improvement for the case $R = 5$. Note that the circles are just touching in this case. Thus, there is little interaction among the groups, and hence little opportunity to improve performance by coordinating transmission schedules.

Next, consider Fig. 6 (for $R = 10$). Again, throughput decreases when β or P_{noise} increases. Because the circles overlap significantly, the impact of interference from members of other groups is considerable. There is a significant difference between $T_{\min}$ and $T_{\max}$. Thus, the performance can be improved significantly through the appropriate choice of transmission schedules. We observe that the variation in throughput is noticeably larger than the previous case, especially for $\beta \geq 1$.

Finally, consider Fig. 7 (for $R = 20$). Because the circles overlap greatly, the interaction among the groups also greatly increases. We observe that the variation in throughput is much larger than the previous 2 cases, especially for $\beta \geq 1$. Thus, schedule optimization significantly improves the performance in this case.

β	T_{TDMA}	$T_{\max}$	T_{ave}	$T_{\min}$	$\frac{T_{\max}}{T_{\text{ave}}}$	$\frac{T_{\max}}{T_{\min}}$	$\frac{T_{\text{ave}}}{T_{\min}}$
$P_{\text{noise}} = 0$
0.1	0.2500	0.9848	0.9823	0.9792	1.002	1.006	1.003
0.2	0.2500	0.9699	0.9653	0.9594	1.005	1.011	1.006
0.5	0.2500	0.9275	0.9178	0.9058	1.011	1.024	1.013
1	0.2500	0.8634	0.8486	0.8314	1.017	1.039	1.021
2	0.2500	0.7559	0.7379	0.7193	1.024	1.051	1.026
5	0.2500	0.5407	0.5279	0.5148	1.024	1.050	1.025
10	0.2500	0.3680	0.3501	0.3358	1.051	1.096	1.043
$P_{\text{noise}} = 0.001$
0.1	0.2478	0.9760	0.9736	0.9705	1.002	1.006	1.003
0.2	0.2455	0.9527	0.9483	0.9427	1.005	1.011	1.006
0.5	0.2391	0.8875	0.8787	0.8681	1.010	1.022	1.012
1	0.2288	0.7921	0.7802	0.7667	1.015	1.033	1.018
2	0.2100	0.6411	0.6301	0.6192	1.017	1.035	1.018
5	0.1651	0.3895	0.3761	0.3662	1.036	1.064	1.027
10	0.1158	0.2161	0.1998	0.1843	1.082	1.172	1.084

Fig. 5 Throughput ($R = 5$)

β	T_{TDMA}	$T_{\max}$	T_{ave}	$T_{\min}$	$\frac{T_{\max}}{T_{\text{ave}}}$	$\frac{T_{\max}}{T_{\min}}$	$\frac{T_{\text{ave}}}{T_{\min}}$
$P_{\text{noise}} = 0$							
0.1	0.2500	0.9103	0.8728	0.8407	1.043	1.083	1.038
0.2	0.2500	0.8343	0.7918	0.7536	1.054	1.107	1.051
0.5	0.2500	0.6665	0.6326	0.5996	1.054	1.112	1.055
1	0.2500	0.5142	0.4816	0.4510	1.068	1.140	1.068
2	0.2500	0.3667	0.3274	0.2812	1.120	1.304	1.164
5	0.2500	0.2096	0.1600	0.1052	1.309	1.992	1.521
10	0.2500	0.1194	0.0777	0.0368	1.538	3.249	2.113
$P_{\text{noise}} = 0.001$							
0.1	0.2376	0.8653	0.8311	0.8029	1.041	1.078	1.035
0.2	0.2261	0.7552	0.7208	0.6899	1.048	1.095	1.045
0.5	0.1961	0.5335	0.5124	0.4908	1.041	1.087	1.044
1	0.1579	0.3644	0.3336	0.3053	1.092	1.193	1.093
2	0.1089	0.2154	0.1796	0.1352	1.199	1.593	1.328
5	0.0486	0.0884	0.0581	0.0251	1.522	3.525	2.317
10	0.0222	0.0449	0.0220	0.0047	2.043	9.638	4.717

Fig. 6 Throughput ($R = 10$)

β	T_{TDMA}	$T_{\max}$	T_{ave}	$T_{\min}$	$\frac{T_{\max}}{T_{\text{ave}}}$	$\frac{T_{\max}}{T_{\min}}$	$\frac{T_{\text{ave}}}{T_{\min}}$
$P_{\text{noise}} = 0$							
0.1	0.2500	0.7034	0.6330	0.5683	1.111	1.238	1.114
0.2	0.2500	0.5839	0.5119	0.4485	1.141	1.302	1.142
0.5	0.2500	0.4121	0.3415	0.2798	1.207	1.473	1.221
1	0.2500	0.2905	0.2221	0.1579	1.308	1.840	1.407
2	0.2500	0.1942	0.1264	0.0684	1.536	2.838	1.848
5	0.2500	0.1018	0.0479	0.0155	2.127	6.577	3.093
10	0.2500	0.0624	0.0199	0.0039	3.140	16.03	5.104
$P_{\text{noise}} = 0.001$							
0.1	0.1824	0.5277	0.4807	0.4426	1.098	1.192	1.086
0.2	0.1397	0.3695	0.3207	0.2824	1.152	1.308	1.135
0.5	0.0755	0.1884	0.1465	0.1076	1.286	1.752	1.362
1	0.0381	0.1006	0.0670	0.0384	1.501	2.622	1.748
2	0.0189	0.0597	0.0297	0.0090	2.007	6.667	3.322
5	0.0100	0.0311	0.0096	0.0009	3.251	36.11	11.11
10	0.0048	0.0120	0.0026	8.1e-5	4.607	147.7	32.06

Fig. 7 Throughput ($R = 20$)

6. SUMMARY

The packetized wireless network considered in this paper operates under the power-capture principle, as well as under realistic conditions such as receiver noise, fading, and other-user interference. Our proposed *parallel* method is a simple method for accomplishing the transmissions between the nodes in the network. Under this method, groups of nodes simply operate in parallel, according to a particular chosen schedule. Our performance analysis of the method shows the impact of schedule, channel fading, receiver noise, and interference on throughput performance. Our parallel method creates opportunities for network performance optimization. In particular, for given channel statistics and topology configurations, the network performance can be significantly improved when nodes coordinate among themselves to operate according to an optimal schedule.

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